This is the author's version of the paper. First published as:

Lesh, R., & English, L. D. (2005). Trends in the evolution of models and modelling perspectives on mathematical learning and problem solving. In H. Chick & J. Vincent (Eds.), Proceedings of the 29th Annual Conference of the International Group for the Psychology of Mathematics Education. (pp. 192-196). University of Melbourne.

TRENDS IN THE EVOLUTION OF MODELS & MODELING PERSPECTIVES ON MATHEMATICAL LEARNING AND PROBLEM SOLVING

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Models and modeling (M&M) research often investigates the nature of understandings and abilities that are needed in order for students to be able to use what they have (presumably) learned in the classroom in "real life" situations beyond school. Nonetheless, M&M perspectives evolved out of research on concept development more than research on problem solving; and, rather than being preoccupied with the kind of word problems emphasized in textbooks and standardized tests, we focus on (simulations of) problem solving "in the wild." Also, we give special attention to the fact that, in a technology-based *age of information*, significant changes are occurring in the kinds of "mathematical thinking" that is coming to be needed in the everyday lives of ordinary people in the 21st century – as well as in the lives of productive people in future-oriented fields that are heavy users of mathematics, science, and technology.

In modern knowledge economies, systems – ranging from communication systems to economic or accounting systems - are among the most important "things" that impact the lives of ordinary people. Some of these systems occur naturally, while others are created by humans. But, in any case, mathematics is useful for making (or making sense of) such systems precisely because mathematics is the study of structure. That is, it is the study of systemic properties of structurally interesting systems.

In future-oriented fields that range from design sciences to life sciences, industry advisors to university programs consistently emphasize that:

> The kind of people we most want to hire are those who are proficient at (a) making sense of complex systems, (b) working within teams of diverse specialists, (c) adapting rapidly to a variety of rapidly evolving conceptual tools, (d) working on multi-staged projects that require planning and collaboration among many levels and types of participants, and (e) developing sharable and re-useable conceptual tools that usually need to draw on a variety of disciplines – and textbook topic areas.

Both of the preceding trends shift attention beyond *mathematics as computation* toward *mathematics as conceptualization, description, and explanation*. But, they also raise the following kinds of questions that lie at the heart of M&M research in mathematics education.

- What is the nature of the most important classes of problem-solving situations where mathematics, science, and technology are needed for success in real life situations beyond school?

- What mathematical constructs or conceptual systems provide the best foundations for success in these situations?
- What does it mean to "understand" these constructs and conceptual systems?
- How do these understandings develop?
- What kinds of experiences facilitate (or retard) development?
- How can people be identified whose exceptional abilities do not fit the narrow and shallow band of abilities emphasized on standardized tests – or even school work?

Related questions are: (a) Why do students who have histories of getting A's on tests and coursework often do not do well beyond school? (b) What is the relationship between the learning of "basic skills" and a variety of different kinds of deeper or higher-order understandings or abilities? (c) Why do problem solving situations that involve collaborators and conceptual tools tend to create as many conceptual difficulties as they eliminate? (d) In what ways is "mathematical thinking" becoming more multi-media - and more contextualized (in the sense that knowledge and abilities are organized around experience as much as around abstractions, and in the sense that relevant ways of thinking usually need to draw on ways for thinking that seldom fall within the scope of a single discipline or textbook topic area). (e) How can instruction and assessment be changed to reflect the fact that, when you recognize the importance of a broader range of understandings and abilities, a broader range of people often emerge as having exceptional potential?

M&M perspectives assume that such questions should be investigated through research, not simply resolved though political processes - such as those that are emphasized when "blue ribbon" panels of experts develop curriculum standards for teaching or testing. Furthermore, we believe that such questions are not likely to be answered through content-independent investigations about *how people learn* or *how people solve problems*, and they are only indirectly about the nature (and/or theRF04 PME29 — 2005 1- 193 development) of humans - or the functioning of human brains. This is because they are about the nature of mathematical and scientific knowledge, and they are about the ways this knowledge is useful in "real life" situations. So, researchers with broad and deep expertise in mathematics and science should play significant roles in collaborating with experts in the learning and cognitive sciences.

Theoretical perspectives for M&M research trace their lineage to modern descendents of Piaget and Vygotsky - but also (and just as significantly) to *American Pragmatists* such as William James, Charles Sanders Peirce, Oliver Wendell Holmes, George Herbert Mead, and John Dewey. And partly for this reason, M&M perspectives reflect "blue collar" approaches to research. That is, we focus on the development of knowledge (and conceptual tools) to inform "real life" decision-making issues – where (a) the criteria for success are not contained within any preconceived theory, (b) productive ways of thinking usually need to draw on more than a single theory, and (c) useful knowledge usually needs to be expressed in the context of conceptual tools that are powerful (for some specific purpose), sharable (with other people), and re-useable (beyond the context in which they were developed). Thus, M&M research often focuses on model-development rather than proceeding too quickly to theory development and hypothesis testing; and, before rushing ahead to try to teach or test various mathematical concepts, processes, beliefs, habits of mind, or components of a productive problem solving personae, we conduct developmental investigations about the nature of what it means to "understand" them.

One way that mathematics educators have investigated questions about what is needed for success beyond school is by observing people "thinking mathematically" in everyday situations. Sometimes, such studies compare "experts" with "novices" who are working in fields such as engineering, agriculture, medicine, or business management - where "mathematical thinking" often is critical for success. Such ethnographic investigations often have been exceedingly productive and illuminating. Nonetheless, from the perspectives of M&M research, they also tend to have some significant shortcomings. For example, we must be skeptical of observations which depend heavily on preconceived notions about where to observe (in grocery stores? carpentry shops? car dealerships? engineering firms? Internet cafés?), whom to observe (street vendors? shoppers? farmers? cooks? engineers? baseball fans?), when to observe (when they're estimating sizes? calculating with numbers? minimizing routes? describing, explaining, or predicting the behaviors of complex systems?), and what to count as "mathematical thinking" (e.g., planning, monitoring, assessing, explaining, justifying steps during multi-step projects, or deciding what information to collect about specific decision-making issues). Consequently, in simple observational studies, close examinations of underlying assumptions often expose unwarranted prejudices about what it means to "think mathematically" - and about the nature of "real life" situations in which mathematics is useful.

A second way to investigate *what's needed for success beyond school* is to use *multitier design experiments* (Lesh, 2002) in which (a) students develop models for making senses of mathematical problem solving situations, (b) teachers develop models for creating (and making sense of) students' modeling activities, and (c) researchers develop models for creating (or making sense of) interactions among students, teachers, and relevant learning environments. We sometimes refer to such studies as *evolving expert studies* (Lesh, Kelly & Yoon, in press) because the final products that are produced tend to represent significant extensions or revisions in the thinking of each of the participants who were involved. Such methodologies respect the opinions of diverse groups of stake holders whose opinions should be considered. On the one hand, nobody is considered to have privileged access to the truth – including, in particular, the researchers. All participants (from students to teachers to researchers) are considered to be in the model development business; and, similar principles are assumed to apply to "scientific inquiry" at all levels. So, everybody's ways of thinking are subjected to examination and possible revision.

For the preceding kind of *three-tiered design experiments*, each tier can be thought of as *a longitudinal development study in a conceptually enriched environment*. That is, a goal is to go beyond studies of typical development in natural environments to also focus on induced development within carefully controlled environments. Finally, because the goal of M&M research is to investigate the nature and development of constructs or conceptual systems (rather than investigating and making claims students per se), we often investigate how understandings evolve in the thinking of "problem solvers" who are in fact teams (or other learning communities) rather than being isolated individuals. So, we often compare individuals with groups in somewhat the same manner that other styles of research might compare experts and novices, or gifted students and average ability students.

Investigations from an M&M perspective have led to the growing realization that, in a technology-based age of information, even the everyday lives of ordinary people are increasingly impacted by systems that are complex, dynamic, and continually adapting; and, this is even more true for people in fields that are heavy users of mathematics and technology. Such fields include design sciences such as engineeringor architecture, social sciences such as economics or business management, or life sciences such as new hyphenated fields involving bio-technologies or nanotechnologies. In such fields, many of the systems that are most important to understand and explain are dynamic (living), self-organizing, and continually adapting.

M&M research is showing that it is possible for average ability students to develop powerful models for describing complex systems that depend on only new uses of elementary mathematical concepts that are accessible to middle school students. However, when we ask *What kind of mathematical understandings and abilities should students master?* attention should shift beyond asking *What kind of computations can they execute correctly?* to also ask *What kind* *of situations can they describe productively? ...* This observation is the heart of M&M perspectives on learning and problem solving.

Traditionally, problem solving in mathematics education has been defined as getting from givens to goals when the path is not obvious. But, according to M&M perspectives, goal directed activities only become problematic when the "problem solver" (which may consist of more than an isolated individual) needs to develop a more productive way of thinking about the situation (given, goals, and possible solution processes). So, solutions to non-trivial problems tend to involve a series of modeling cycles in which current ways of thinking are iteratively expressed, tested, and revised; and, each modeling cycle tends to involve somewhat different interpretations of givens, goals, and possible solution steps.

Results from M&M research make it clear that average ability students are indeed capable of developing powerful mathematical models and that the constructs and conceptual systems that underlie these models often are more sophisticated than anything that anybody has tried to teach the relevant students in school.

However, the most significant conceptual developments tend to occur when students are challenged to repeatedly express, test, and revise their own current ways thinking - not because they were guided along a narrow conceptual trajectory toward (idealized versions of) their teachers ways of thinking (Lesh & Yoon, 2004). That is, development looks less like progress along a path; and, it looks more like an inverted genetic inheritance tree - where great grandchildren trace their evolution from multiple lineages which develop simultaneously and interactively.

In general, when knowledge develops through modeling processes, the knowledge and conceptual tools that develop are instances of situated cognition. Models are always molded and shaped by the situations in which they are created or modified; and, the understandings that evolve are organized around experience as much as around abstractions. Yet, the models and underlying conceptual systems that evolve often represent generalizable ways of thinking. That is, they are not simply situation specific knowledge which does not transfer. This is because models (and other conceptual tools) are seldom worthwhile to develop unless they are intended to by powerful (for a specific purpose in a specific situation), re-useable (in other situations), and sharable (with other people).

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